Unit 1:
Algebra and Numbers
Part A

Outcome AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors; greatest common factor; least common multiple; square root; and cube root.

(this outcome will be covered in Sections 3.1 and 3.2)

3.1 Factors and Multiples of Whole Numbers

LESSON FOCUS	Determine prime factors, greatest common factors, and least common multiples of whole numbers.

Make Connections

In these belts, the patterns are 12 beads long and 40 beads long. How many beads long must a belt be for it to be created using either pattern?

Activate Prior Learning:

Factors and Multiples

A factor is a number that divides exactly into another number.

What are the factors of 30?

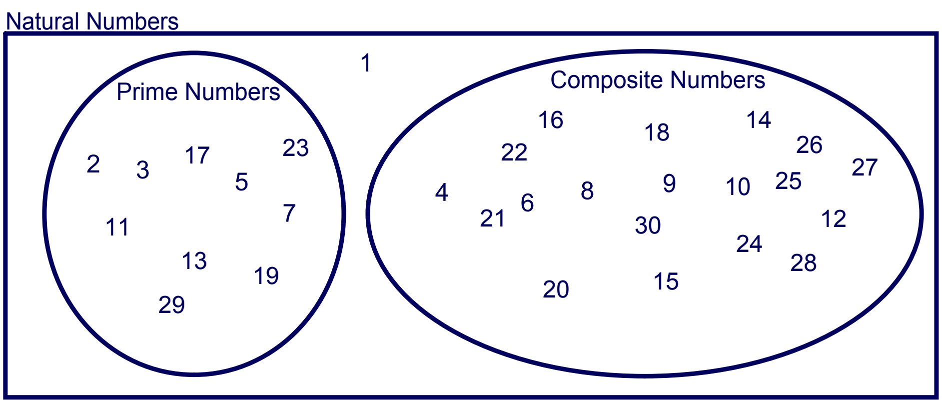
The multiples of a number are determined by multiplying the number by 1, 2, 3, 4, and so on, or by skip counting.

What are some multiples of 12?

Activate Prior Learning:

Prime Numbers

A prime number has exactly 2 factors, 1 and itself.
A composite number has more than 2 factors.
Sort these numbers.



TRY THIS

Work with a partner.

A. List some powers of 2. Make another list of powers of 3. Pick a number from each list and multiply them to create a different number. What are the factors of this number? What are some multiples of this number?

B. Compare your number to your partner's number. Which factors do the two numbers have in common? Which factor is the greatest?

C. What are some multiples the two numbers have in common? Which multiple is the least?

D. How can you use the product of powers from Step A to determine the greatest factor and the least multiple that the numbers have in common?

Powers of 2:

2, 4, 8

2^1 2^2 2^3 — $2 \cdot 2 \cdot 2$

$4 \cdot 9$

36

Powers of 3:

3, 9, 27

Complete the table.

Product of a power of 2 and a power of 3	Factors	Multiples
36	1, 2, 3, 6, 12, 18, 36	72, 108

Circle the factors that the two numbers have in common.
Which factor is the greatest?

Circle the multiples that the two numbers have in common.
Which multiple is the least?

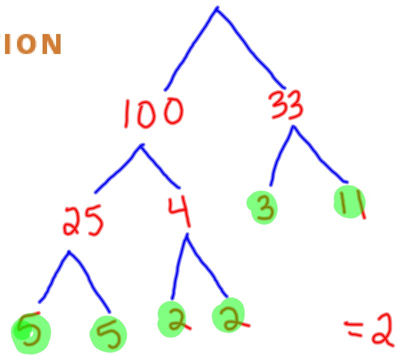
How can you use the product of powers to determine the greatest factor and the least multiple that the numbers have in common?

Example 1 Determining the Prime Factors of a Whole Number

Math_Rocks!

Write the prime factorization of 3300.

SOLUTION



$= 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

$= 2^2 \cdot 3 \cdot 5^2 \cdot 11$

CHECK YOUR UNDERSTANDING

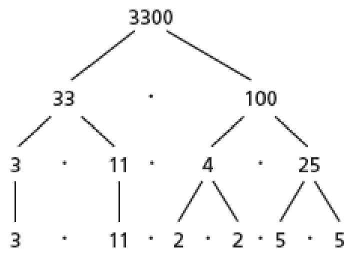
Example 1 Determining the Prime Factors of a Whole Number

Write the prime factorization of 3300.

SOLUTIONS

Method 1

Draw a factor tree.
Write 3300 as a product of 2 factors.
Both 33 and 100 are composite numbers, so we can factor again.
Both 3 and 11 are prime factors, but 4 and 25 can be factored further.



The prime factors of 3300 are 2, 3, 5, and 11.
The prime factorization of 3300 is: $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$,
or $2^2 \cdot 3 \cdot 5^2 \cdot 11$

(Solution continues.)

Example 1 Determining the Prime Factors of a Whole Number

Method 2

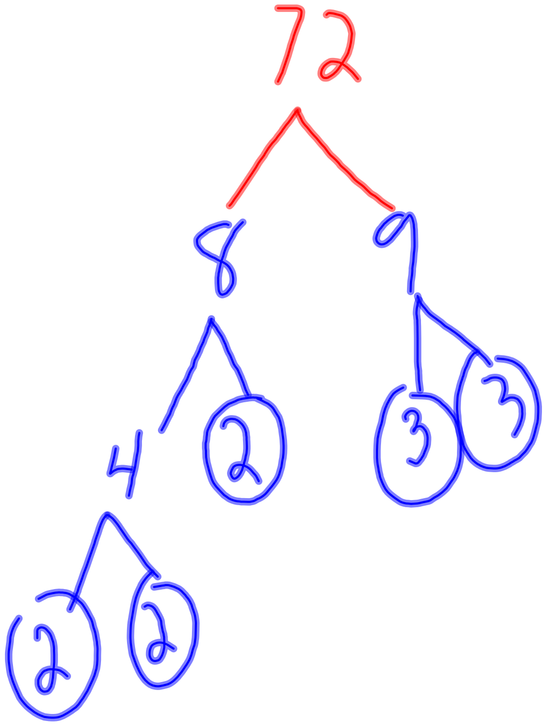
Use repeated division by prime factors.
Begin by dividing 3300 by the least prime factor, which is 2.
Divide by this prime factor until it is no longer a factor.
Continue to divide each quotient by a prime factor until the quotient is 1.

$3300 \div 2 = 1650$
$1650 \div 2 = 825$
$825 \div 3 = 275$
$275 \div 5 = 55$
$55 \div 5 = 11$
$11 \div 11 = 1$

The prime factors of 3300 are 2, 3, 5, and 11.

The prime factorization of 3300 is: $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$,
or $2^2 \cdot 3 \cdot 5^2 \cdot 11$

CHECK YOUR UNDERSTANDING



$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$2^3 \cdot 3^2$

Warm-up #1

Date: 10

#1 What is a factor? 1

#2 What are the factors of the number 16? 2

#3 Tell me 3 multiples of the number 12. 3

#4 Draw a factor tree for the number 40 4

Warm-up #1

Date:

#1 What is a factor?

A factor is a number that divides exactly into another number.

#2 What are the factors of the number 16?

1, 2, 4, 8, 16

#3 Tell me 3 multiples of the number 12.

12, 24, 36, 48

#4 Draw a factor tree for the number 40

40
4 10
2 2 2 5

$2 \cdot 2 \cdot 2 \cdot 5$

$= 2^3 \cdot 5$

$\frac{}{10}$

+1

4

factors

multiples

prime numbers

2 factors → 1 + itself

Ex. 3
1, 3

composite number

→ more than 2 factors Ex. 6
1, 2, 3, 6

factor trees

24
2 12
2 6
3 2

$= 2 \cdot 2 \cdot 2 \cdot 3$

$= 2^3 \cdot 3$

prime factorization

1 This expression is the prime factorization of what number?

A 300

B ~~30~~

C ~~120~~

$2^2 \cdot 3 \cdot 5^2$

$4 \cdot 3 \cdot 25$

2 Is this expression the prime factorization of 72?

A Yes

B No

$3 \cdot 3$

$3^2 \cdot 8$

$9 \cdot 8$

72

When a factor of a number has exactly two divisors, 1 and itself, the factor is a *prime factor.*

Factors of 12: 1, 2, 3, 4, 6, 12

?

?

?

?

?

The **prime factorization** of a natural number is the number written as a product of its prime factors.

?

?

3.1 Factors and Multiples of Whole Numbers

Are the numbers 0 and 1 considered prime numbers? Explain.

<u>Possible Answer</u>:
A prime number is described as any number that has only has two (positive whole number) factors - itself and 1.

2 is a prime number, since its only factors are 1 and 2.
47 is a prime number, because its only factors are 1 and 47.

1 has only one factor - itself, 1. You multiply 1 x 1 to get 1, and you can't exactly treat 1 and 1 as separate factors.

0 is a similar story, since it could technically be seen as a number with infinite factors. All you have to do is multiply a number by 0, and you will get 0.
3 x 0 = 0
0 x 41 = 0
144 x 0 = 0
21,373,483,879,182,994,121,677,325 x 0 = 0

For 2 or more natural numbers, we can determine their **greatest common factor**.

Example 2 Determining the Greatest Common Factor

Determine the greatest common factor of 138 and 198.

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Determining the Greatest Common Factor

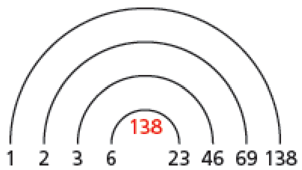
Determine the greatest common factor of 138 and 198.

SOLUTIONS

Method 1

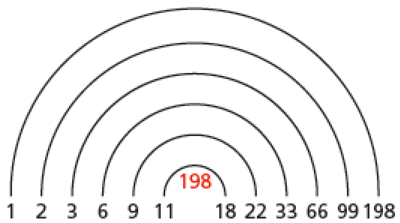
Use division facts to determine all the factors of each number.
Record the factors as a "rainbow."

$138 \div 1 = 138$
$138 \div 2 = 69$
$138 \div 3 = 46$
$138 \div 6 = 23$



Since 23 is a prime number, there are no more factors of 138.

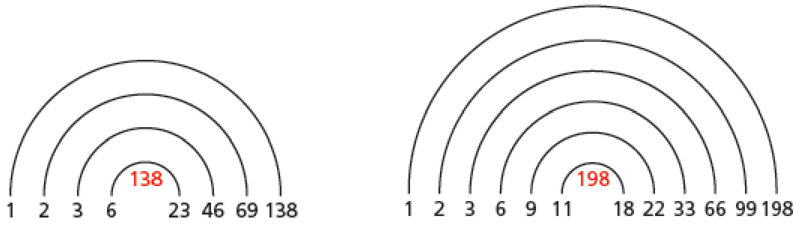
$198 \div 1 = 198$
$198 \div 2 = 99$
$198 \div 3 = 66$
$198 \div 6 = 33$
$198 \div 9 = 22$
$198 \div 11 = 18$



There are no more factors of 198 between 11 and 18.

(Solution continues.)

Example 2 Determining the Greatest Common Factor



The common factors of 138 and 198 are: 1, 2, 3, and 6.
So, the greatest common factor is 6.

Method 2

Check to see which factors of 138 are also factors of 198.
Start with the greatest factor.
The factors of 138 are: 1, 2, 3, 6, 23, 46, 69, 138
198 is not divisible by 138, 69, 46, or 23.
198 is divisible by 6: $198 \div 6 = 33$
The greatest common factor is 6.

(Solution continues.)

Example 2 Determining the Greatest Common Factor

Method 3 (Best way!)

Write the prime factorization of each number.
Highlight the factors that appear in each prime factorization.

$138 = 2 \cdot 3 \cdot 23$

$198 = 2 \cdot 3 \cdot 3 \cdot 11$

The greatest common factor is $2 \cdot 3$, which is 6.

GCF?

46 → 2, 23

2 · 23

GCF = 2

84 → 2, 42 → 2, 21 → 3, 7

2 · 2 · 3 · 7

CHECK YOUR UNDERSTANDING

Tuesday, September 13th, 2011

- Check Homework (Pg. 140) Please have it opened at your desk for me to see.
- Learn/review the lowest common multiple (LCM)
- Some notes on Section 3.1
- Classwork/Homework

Please Note: There will be a quiz this Friday, September 16 on Sections 3.1 and 3.2 combined. If you need extra help please come see me to schedule a time that we can both meet or stop by at lunch hour.

Please complete the following question while I come around to check your homework. Have your homework opened at your desk before I get there.

1. Write the prime factorization of 2646.

6   441

7   378

2   1323

7×54

2×27

$3 \times 9 \rightarrow 3 \times 3$

$7^2 \cdot 2^1 \cdot 3^3$

3.1 Factors and Multiples of Whole Numbers

Classwork/Homework

Please note that any work not completed during class time given is expected to be completed for homework. I may check tomorrow to see that the work was completed. Marks are based on completion (I want to see that you tried your best on every question), not on accuracy (it's ok if you didn't get the answer exactly right!). Therefore, try your best, and make notes of where you are having difficulty!

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#3, 4ac, 5df, 6ac, 8ab, 9a

3. List the first 6 multiples of each number.

a) 6 **b)** 13 **c)** 22
d) 31 **e)** 45 **f)** 27

a) 6, 12, 18, 24, 30, 36

4. List the prime factors of each number.

a) 40 **b)** 75 **c)** 81
d) 120 **e)** 140 **f)** 192

a) 40

4 10

2 2 2 5

$= 2^3 \cdot 5$

5. Write each number as a product of its prime factors.

a) 45 **b)** 80 **c)** 96
d) 122 **e)** 160 **f)** 195

d) 122

2 61

$2 \cdot 61$

f) 195

5 39

13 3

$3 \cdot 5 \cdot 13$

6. Use powers to write each number as a product of its prime factors.

a) 600 b) 1150
c) 1022 d) 2250
e) 4500 f) 6125

7. Explain why the numbers 0 and 1 have no prime factors.

8. Determine the greatest common factor of each pair of numbers.

a) 46, 84 b) 64, 120
c) 81, 216 d) 180, 224
e) 160, 672 f) 220, 860

9. Determine the greatest common factor of each set of numbers

a) 150, 275, 420 5 b) 120, 960, 1400
c) 126, 210, 546, 714 d) 220, 308, 484, 988

600
5 120
12 10
6 2 5 2
3 2
$2^3 \cdot 3 \cdot 5^2$

8a) 46 → 1, 2, 23, 46 GCF = 2
84 →

GCF

64
8 8
4 2 4 2
2 2 2 2
$\boxed{2^6}$

120
12 10
6 2 2 5
3 2
$\boxed{2^3 \cdot 3 \cdot 5}$

GCF $\boxed{2^3}$ = 8

CHECK YOUR UNDERSTANDING

2. Determine the greatest common factor of 126 and 144.

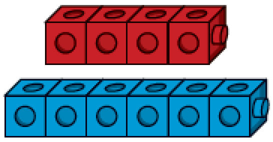
handout cubes

To generate multiples of a number, multiply the number by the natural numbers; that is, 1, 2, 3, 4, 5, and so on. For example, some multiples of 26 are:

$26 \cdot 1 = 26 \qquad 26 \cdot 2 = 52 \qquad 26 \cdot 3 = 78 \qquad 26 \cdot 4 = 104$

For 2 or more natural numbers, we can determine their **least common multiple.**

We can determine the least common multiple of 4 and 6 by combining identical copies of each smaller chain to create two chains of equal length.



→ ? ? ?

? ?

?

LCM

4 and 6

4, 8, 12, 16

6, 12, 18

LCM = 12

12 and 14

<u>LCM</u> $= \underline{84}$

12, 24, 36, 48, 60, 72, 84

38

19 2

$= \boxed{2 \cdot 19}$

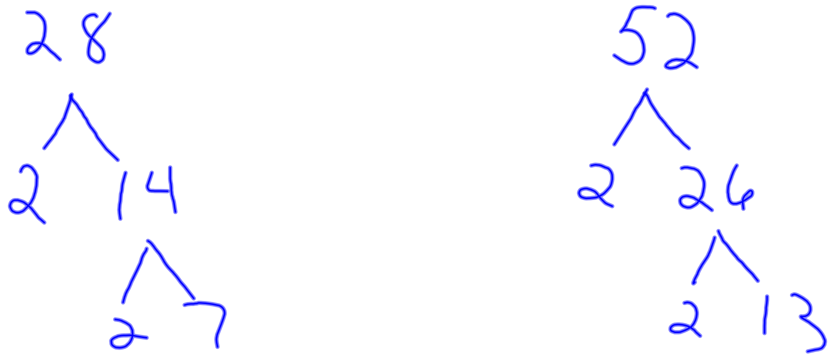
42

2 21

3 7

$\boxed{2 \cdot 3 \cdot 7}$

$LCM = 2 \cdot 3 \cdot 7 \cdot 19 = 798$



$2^2 \cdot 7$ $\qquad$ $2^2 \cdot 13$

$$\text{LCM} = 2^2 \cdot 7 \cdot 13$$
$$= 4 \cdot 7 \cdot 13 = \boxed{364}$$

Example 3 Determining the Least Common Multiple

Determine the least common multiple of 18, 20, and 30.

SOLUTION

CHECK YOUR UNDERSTANDING

Example 3 Determining the Least Common Multiple

Determine the least common multiple of 18, 20, and 30.

SOLUTIONS

Method 1

List the multiples of each number until the same multiple appears in all 3 lists.

Multiples of 18 are: 18, 36, 54, 72, 90, 108, 126, 144, 162, **180**, …
Multiples of 20 are: 20, 40, 60, 80, 100, 120, 140, 160, **180**, …
Multiples of 30 are: 30, 60, 90, 120, 150, **180**, …

The least common multiple of 18, 20, and 30 is 180.

Method 2

Check to see which multiples of 30 are also multiples of 18 and 20.

The multiples of 30 are: 30, 60, 90, 120, 150, 180, …

18 divides exactly into: 90, **180**, …
20 divides exactly into: 60, 120, **180**, …

The least common multiple of 18, 20, and 30 is 180.

(Solution continues.)

Example 3 Determining the Least Common Multiple

Method 3

Write the prime factorization of each number.
Highlight the greatest power of each prime factor in any list.

$18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$ The greatest power of 3 in any list is 3^2.
$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$ The greatest power of 2 in any list is 2^2.
$30 = 2 \cdot 3 \cdot 5$ The greatest power of 5 in any list is 5.

The least common multiple is the product of the greatest power of each prime factor:

$$2^2 \cdot 3^2 \cdot 5 = 4 \cdot 9 \cdot 5$$
$$= 180$$

The least common multiple of 18, 20, and 30 is 180.

Wednesday, September 14th, 2011

- Take out your notes from yesterday and finish copying them.
- Review word problems with GCF and LCM
- Do some practice questions
- Begin Section 3.2
- Notes/Examples
- Practice Questions

Please Note: There will be a quiz this Friday, September 16 on Sections 3.1 and 3.2 combined. If you need extra help please come see me to schedule a time that we can both meet or stop by at lunch hour.

NOTES

To factor a whole number it is helpful to express a number as a product of its **prime factors**. As **prime numbers** these factors have exactly two divisors, 1 and itself.

- The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
- Natural numbers greater than 1 that are not prime are called **composite numbers**.

As an example of **prime factorization**, 24 can be expressed as a product of its prime factors: $24 = 2 \times 2 \times 2 \times 3$, or $24 = 2^3 \times 3$.

- A **factor tree** can be used to determine a number's prime factors.
- To avoid confusion with the variable x, we use a dot to indicate multiplication as in: $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

The **greatest common factor (GCF)** of two or more numbers is the greatest factor that the two numbers have in common.

- For two numbers, one way to determine the GCF is to identify the prime factors common to both numbers, and then to take the product of these factors.
- For example, for 60 and 24, $60=2\cdot2\cdot3\cdot5$, and $24=2\cdot2\cdot2\cdot3$. Multiplying the factors in common gives: $2\cdot2\cdot3=12$, so 12 is the GCF.

The **least common multiple or LCM** is the smallest number that is a multiple of two or more numbers

- One method is to compare multiples of each number until a common multiple is found. For example, the multiples of 6 are 6, 12, 18, 24, 30, and the multiples of 10 are 10, 20, 30. The first multiple they have in common is 30, so this is the LCM.
- Another method is to do the prime factorization of each number and then find the product of the greatest power of each prime factor.

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LCM

#11 b) 15 32 44

15: 3, 5

32: 16, 2; 16: 4, 4; 4: 2, 2; 4: 2, 2

44: 4, 11; 4: 2, 2

$3 \cdot 5$

2^5

$2^2 \cdot 11$

$LCM = 2^5 \cdot 3 \cdot 5 \cdot 11$
$= 32 \cdot 3 \cdot 5 \cdot 11$
$= 5280$

Reducing fractions

$\frac{4}{8}$

GCF = 4

#15 find the GCF

$\frac{650}{900}$

factor tree

#16 LCM

e) $\frac{9}{25} + \frac{7}{15} - \frac{5}{8}$

$$\frac{9}{25} \xrightarrow{\times 24} \frac{216}{600}, \quad \frac{7}{15} \xrightarrow{\times 40} \frac{280}{600}$$

$$= \frac{216}{600} + \frac{280}{600} - \frac{375}{600} = \frac{121}{600}$$

LCM

$25 = 5^2$

$25 \to 5, 5$

$15 = 3 \cdot 5$

$15 \to 3, 5$

$8 = 2^3$

$8 \to 4, 2$; $4 \to 2, 2$

$$LCM = 2^3 \cdot 3 \cdot 5^2$$
$$= 8 \cdot 3 \cdot 25$$
$$= 600$$

Discuss the Ideas

How can you use the prime factorization of a number to determine all the factors of that number?

Use the number 30 for an example

3.1 Factors and Multiples of Whole Numbers

Example 4 Solving Problems that Involve Greatest Common Factor and Least Common Multiple

a) What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.

b) What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.

SOLUTION

CHECK YOUR UNDERSTAN

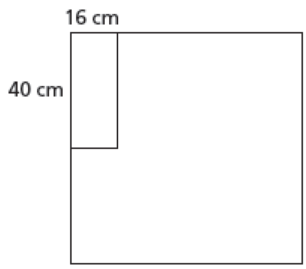
Example 4 Solving Problems that Involve Greatest Common Factor and Least Common Multiple

a) What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.

b) What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.

SOLUTION

a) In the square, arrange all the rectangles with the same orientation.



The shorter side of each rectangle measures 16 cm.
So, the side length of the square must be a multiple of 16.

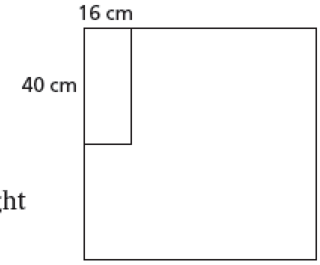
The longer side of each rectangle measures 40 cm.
So, the side length of the square must be a multiple of 40.

(Solution continues.)

3.1 Factors and Multiples of Whole Numbers

Example 4 Solving Problems that Involve Greatest Common Factor and Least Common Multiple

So, the side length of the square must be a common multiple of 16 and 40.

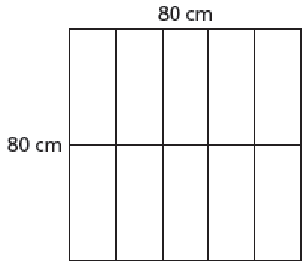


The side length of the smallest square will be the least common multiple of 16 and 40.

Write the prime factorization of each number. Highlight the greatest power of each prime factor in either list.

$16 = 2^4$
$40 = 2^3 \cdot 5$

The least common multiple is:
$2^4 \cdot 5 = 80$

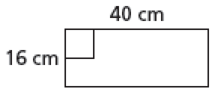


The side length of the smallest square is 80 cm.

(Solution continues.)

Example 4 Solving Problems that Involve Greatest Common Factor and Least Common Multiple

b) The shorter side of the rectangle measures 16 cm. So, the side length of the square must be a factor of 16.



The longer side of the rectangle measures 40 cm. So, the side length of the square must be a factor of 40.

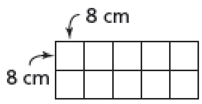
So, the side length of the square must be a common factor of 16 and 40.

The side length of the largest square will be the greatest common factor of 16 and 40.

Write the prime factorization of each number. Highlight the prime factors that appear in both lists.

$16 = 2 \cdot 2 \cdot 2 \cdot 2$
$40 = 2 \cdot 2 \cdot 2 \cdot 5$

The greatest common factor is:
$2 \cdot 2 \cdot 2 = 8$



The largest square has side length 8 cm.

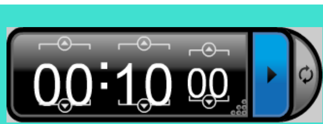
CHECK YOUR UNDERSTANDING

Thursday, September 15th, 2011

- Check and go over homework (Pg.140)
- Notes/Examples on Square roots and Cube roots
- Practice Questions

Please Note: There will be a quiz this Friday, September 16 on Sections 3.1 and 3.2 combined. If you need extra help please come see me to schedule a time that we can both meet or stop by at lunch hour. Today I have duty in the cafeteria second half of lunch, feel free to come there to get help as well.

Classwork/Homework



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#11a, 12, 15ad, 16ce, 18, 19

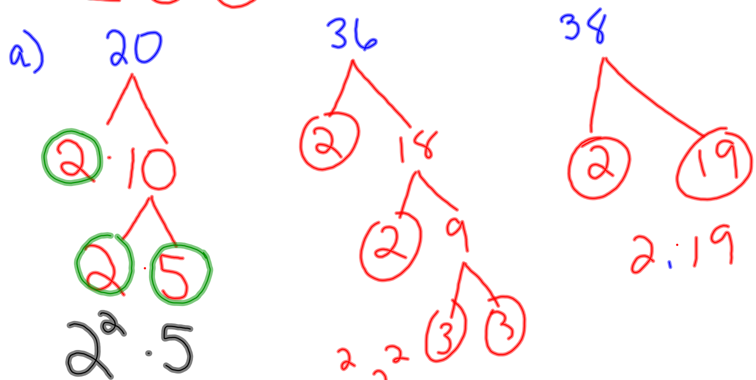
I'm only giving you 10 minutes to work on these now. You may have more time at the end of class. If you don't finish in class, you will need to finish it for homework.

12 14
GCF = 2
LCM =

11. Determine the least common multiple of each set of numbers.

a) 20, 36, 38 b) 15, 32, 44
c) 12, 18, 25, 30 d) 15, 20, 24, 27

12. Explain the difference between determining the greatest common factor and the least common multiple of 12 and 14.



$2^2 \cdot 5$ $2^2 \cdot 3^2$ $2 \cdot 19$

$LCM = 2^2 \cdot 5 \cdot 19 \cdot 3^2$
$= 4 \cdot 5 \cdot 19 \cdot 9$
$= \boxed{3420}$

15. How could you use the greatest common factor to simplify a fraction? Use this strategy to simplify these fractions.

a) $\frac{185}{325}$ b) $\frac{340}{380}$ c) $\frac{650}{900}$

d) $\frac{840}{1220}$ e) $\frac{1225}{2750}$ f) $\frac{2145}{1105}$

GCF = 5

325

$\div 5$

$\frac{185}{325} = \frac{37}{65}$

$\div 5$

16. How could you use the least common multiple to add, subtract, or divide fractions? Use this strategy to evaluate these fractions.

a) $\frac{9}{14} + \frac{11}{16}$ b) $\frac{8}{15} + \frac{11}{20}$

c) $\frac{5}{24} - \frac{1}{22}$ d) $\frac{9}{10} + \frac{5}{14} + \frac{4}{21}$

e) $\frac{9}{25} + \frac{7}{15} - \frac{5}{8}$ f) $\frac{3}{5} - \frac{5}{18} + \frac{7}{3}$

g) $\frac{3}{5} \div \frac{4}{9}$ h) $\frac{11}{6} \div \frac{2}{7}$

#16

c) $\frac{5}{24} - \frac{1}{22} = \frac{55}{264} - \frac{12}{264} = \boxed{\frac{43}{264}}$

$\times 11$ $\times 12$

We need to find a common denominator

So, we need to find the LCM of 24 and 22.

24: 2, 12; 12: 2, 6; 6: 2, 3

$2^3 \cdot 3$

22: 2, 11

$2 \cdot 11$

$LCM = 2^3 \cdot 3 \cdot 11$
$= 8 \cdot 3 \cdot 11$
$= 24 \cdot 11$
$=$

52×11
572

18. Do all whole numbers have at least one prime factor? Explain. Yes, all whole numbers have prime factors except for the number one.

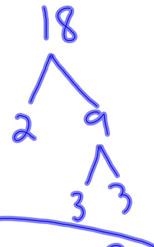
19. **a)** What are the dimensions of the smallest square that could be tiled using an 18-cm by 24-cm tile? Assume the tiles cannot be cut.

LCM

72 by 72

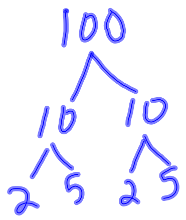
b) Could the tiles in part a be used to cover a floor with dimensions 6.48 m by 15.12 m? Explain.

100 cm = 1 m



$2 \cdot 3 \cdot 3$

$$18 = 2 \cdot 3 \cdot 3$$
$$6 \cdot 3$$
$$2 \cdot 9$$



$$\begin{aligned} 100 &= 2 \cdot 2 \cdot 5 \cdot 5 \\ &= 4 \cdot 25 \\ &= 20 \cdot 5 \\ &= 10 \cdot 10 \end{aligned}$$

Attachments

Math_Rocks!_Factor_Down_a_Tree.webm